Superluminal Quantum Models of the Photon and Electron*

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Abstract

A spatial model of a free electron (or a positron) is formed by a proposed superluminally circulating point-like charged superluminal quantum. The model includes the Dirac equation’s electron spin $\frac{1}{2} \hbar$ and magnetic moment $e\hbar/2m$ as well as three Dirac equation measures of the electron’s Zitterbewegung (jittery motion) described by Schrödinger: a speed of light velocity $c$, a frequency of $2mc^2/h = 2.5 \times 10^{20}$ Hz, and a size of $\frac{1}{2} \hbar/mc = 1.9 \times 10^{-13}$ m. The electron’s superluminal quantum has a closed double-looped helical trajectory whose circular axis’ double-looped length is one Compton wavelength $\hbar/mc$. The electron’s superluminal quantum carries the electron’s charge $-e$ and its energy and momentum and has a photon-like frequency and wavelength along its closed helical trajectory. The superluminal quantum’s maximum speed in the electron model’s rest frame is $2.797c$. In the electron’s rest frame, the equations for the superluminal quantum’s position along its closed double-looped helical trajectory, where the electron’s spin and magnetic moment are along the z-axis, are:

$$\begin{align*}
x(t) &= R_0(1 + \sqrt{2}\cos(\omega_0 t))\cos(2\omega_0 t) \\
y(t) &= R_0(1 + \sqrt{2}\cos(\omega_0 t))\sin(2\omega_0 t) \\
z(t) &= R_0\sqrt{2}\sin(\omega_0 t)
\end{align*}$$

where $R_0 = \frac{1}{2} \hbar/mc$ and $\omega_0 = mc^2/\hbar$

A photon is modeled by an uncharged superluminal quantum moving at $1.414c$ along an open 45-degree helical trajectory with radius $R = \lambda/2\pi$.

Introduction

Ever since the discovery of the electron in 1897 [1] there has been a series of efforts to model the electron by a 3D spatial structure [2,3 and accompanying references]. When an experiment in 1922 [4] led to the measurement of the electron’s magnetic moment of $e\hbar/2mc$, the Bohr magneton, and other experiments led to the discovery [5].
in 1925 of the electron’s spin or angular momentum of \( \frac{1}{2} \hbar \), efforts were made to incorporate these features into electron models. Some of these models required superluminal motion in order to get both the electron’s spin and the magnetic moment correct, and were rejected partly for this reason. None of these electron models were generally accepted, and as a result the spin and magnetic moment of the electron were considered to be intrinsic features of an electron, not cause by its spatial extension or rotation.

Dirac introduced [6] his relativistic theory of the electron in 1928. This work did not include a model of the electron itself, and assumed the electron was a point-like particle. Schrödinger [7] analyzed the results of the Dirac equation for a free electron, and described a high-frequency *Zitterbewegung* motion which appeared to be due to the intereference between positive and negative energy terms in the solution. Barut and Bracken [8] analyzed the Dirac equation and proposed a spatial description of the electron where the *Zitterbewegung* would produce the electron’s spin as the orbital angular momentum of the electron’s internal system, while the electron’s rest mass would be the electron’s internal energy in its rest frame. Hestenes [9,10,11,12] reformulated the Dirac equation through a mathematical approach (Clifford algebra) that brings out a geometric approach to understanding *Zitterbewegung* and to modeling the electron, such as identifying the phase of the Dirac spinor with the spatial angular momentum of the electron.

Despite more than 75 years of modeling the electron with spin, no satisfactory 3D electron model has been developed. The present approach reintroduces superluminality into electron modeling, based on modeling the Dirac electron’s *Zitterbewegung*. This approach combines modeling an electron with modeling a photon in a way that both the photon and the electron have related superluminal structures.

**The superluminal quantum approach to modeling photons and electrons**

In this approach, point-like particles are postulated called superluminal quanta (to distinguish them from elementary particles such as electron and photons.) They have an
associated energy \( E \), with its associated frequency \( f \) and angular frequency \( \omega = 2\pi f \), an
associated momentum \( p \) with its associated wavelength \( \lambda \) and wave number \( k = 2\pi / \lambda \),
and an associated charge (in the case of charged elementary particles). One superluminal
quantum forms a photon or an electron. Superluminal quanta move superluminally in
helical trajectories, which may be open (for an electron) or closed (for a photon).
Movement along its trajectory produces an elementary particle such as an electron or a
photon. The type of helical trajectory the superluminal quantum has determines what
kind of elementary particle, such as photon or electron, is produced by the superluminal
quantum’s motion. So far this approach has only been applied here to photons and
electrons (and positrons), but in principle this approach may be extended to describe
other elementary particles such as quarks and gluons.

Energy is a scalar quantity and is associated directly with the superluminal
quantum. The relationship is \( E = \hbar \omega \) where \( \omega \) is the angular frequency of rotation of the
superluminal quantum along its helical trajectory, whether the helix is open or closed.
Momentum is a vector quantity. The superluminal quantum has its own momentum
vector \( \vec{P} \) directed tangentially along its helical trajectory. But the superluminal quantum’s
quantitative momentum relationship is \( p = \hbar k \), where \( p \) is the component of the
superluminal quantum’s momentum vector \( \vec{P} \) that is projected parallel to the helical axis
around which the superluminal quantum is moving. \( \vec{P} \) itself will vary in direction as the
superluminal quantum moves along its helical trajectory. \( \vec{P} \) will have constant magnitude
when traveling along an open helix which has constant curvature, but will change in
magnitude as well as direction with the changing curvature of its helical trajectory as the
superluminal quantum travels around a closed helix, and its projected magnitude \( p \) and
the its corresponding \( k \) will vary accordingly. The projected velocity of any superluminal
quantum along its helical axis, whether the helix is open or closed, is postulated to always
be the speed of light \( c \). This requires that the superluminal quanta themselves will always
move at superluminal speeds—with constant speed along an open helix with a straight
axis, but with varying speed along an open helix with a curving axis or with a closed
helix.

The following superluminal quantum models of a photon and an electron will
illustrate the superluminal quantum’s properties.
The superluminal quantum model of a photon

A photon is modeled as a superluminal quantum traveling along a straight, open helical trajectory of radius $R$, pitch (wavelength) $\lambda$, and angle $\theta$ with the forward direction. In this helix, these three quantities are related geometrically by $\tan \theta = \frac{2\pi R}{\lambda}$. Using the superluminal quantum model, the result is:

1) The forward helical angle $\theta$ is found to be $45^\circ$, for any photon wavelength.

2) The photon helix’s radius is found to always be $R = \frac{\lambda}{2\pi}$.

3) The speed of the superluminal quantum is $\sqrt{2}c = 1.414c$ along its helical trajectory.

These results are derived below.

The superluminal quantum, with total momentum $\vec{P}$ directed along its helical trajectory, has a forward component of momentum $P\cos(\theta)$ determined by the wavelength $\lambda$ of the helix, and a transverse component of momentum $P\sin(\theta)$ that is
used to calculate the spin of the photon. So the superluminal quantum’s longitudinal component of momentum is

\[ P \cos(\theta) = \frac{h}{L} \]  

(1)

the experimental longitudinal or linear momentum of a photon. The total momentum \( \vec{P} \)’s transverse component of momentum is \( P \sin \theta \). This transverse component is perpendicular to the helical radius vector \( \vec{R} \) to the superluminal quantum from the helical axis. The magnitude \( S \) of the angular momentum or spin of the superluminal quantum model is then

\[ S = R \vec{P} \sin(\theta) = \frac{h}{2\pi} \]  

(2)

the experimental spin or angular momentum of the photon. Combining equations (1) and (2) gives

\[ \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta) = \frac{\lambda}{2\pi R} \]  

(3)

Now look at the helical geometry. As the superluminal quantum advances along the helix a distance \( \lambda \) (one wavelength) in the longitudinal direction, the superluminal quantum travels a transverse distance \( 2\pi R \), i.e. once around the circle of radius \( R \) of the helix. From the way the helix’s angle \( \theta \) is defined,

\[ \tan(\theta) = \frac{2\pi R}{\lambda} \]  

(4)

We now have two equations (3) and (4) for \( \tan(\theta) \). So setting them equal gives

\[ \tan(\theta) = \frac{2\pi R}{\lambda} = \frac{\lambda}{2\pi R} \]  

(5)

This will only be true if

\[ \lambda = 2\pi R \]  

(6)

that is,

\[ R = \frac{\lambda}{2\pi} \]  

(7)

and since \( \tan \theta = 1 \) we have

\[ \theta = 45^\circ \]  

(8)

These results are true for any photon in this superluminal quantum model of the photon. So since \( \theta = 45^\circ \) and the velocity of the superluminal quantum along its helical axis is postulated to be \( c \), then the velocity of the photon model is also \( c \), and the velocity \( v \) of the superluminal quantum along its helical trajectory is

\[ v = \frac{c}{\cos(45^\circ)} = \frac{c}{\sqrt{2}} = c/1.414.. = 1.414..c \]  

(9)
Equations of motion of the photon’s superluminal quantum

According to the direction of the helicity of the superluminal quantum’s helical trajectory, the modeled photon will be right-handed or left-handed circularly polarized. Consider a right handed helix of wavelength $\lambda$ and therefore radius $R = \lambda / 2\pi$. The helix will extend in the z direction centered along the z-axis. Its wave number will be $k = 2\pi / \lambda$. If the superluminal quantum starts at $x = \lambda / 2\pi$, $y = 0$, and $z = 0$ its helical trajectory is given in complex notation by

$$R(z(t)) = Re^{ikz(t)} = \frac{\lambda}{2\pi} e^{\frac{2\pi}{\lambda} z(t)}$$

(10)

that is,

$$x(z(t)) = \frac{\lambda}{2\pi} \cos\left(\frac{2\pi}{\lambda} z(t)\right)$$

$$y(z(t)) = \frac{\lambda}{2\pi} \sin\left(\frac{2\pi}{\lambda} z(t)\right)$$

(11)

$$z(t) = ct$$

while since the angular frequency of the superluminal quantum around the helix is $\omega$, and the superluminal quantum’s longitudinal component of speed along the helical axis is $c$, the position of the superluminal quantum along its helical trajectory starting at $t = 0$ is

$$R(t) = \frac{\lambda}{2\pi} e^{\frac{2\pi c t}{\lambda}} = \frac{\lambda}{2\pi} e^{i\omega t}$$

(12)

since $\frac{2\pi c}{\lambda} = 2\pi f = \omega$. This is the complex notation form of

$$x(t) = \frac{\lambda}{2\pi} \cos(\omega t)$$

$$y(t) = \frac{\lambda}{2\pi} \sin(\omega t)$$

(13)

$$z(t) = ct$$

Note that when the photon’s superluminal quantum travels along its helical trajectory, the helix itself is stationary in space. It does not move forward with velocity $c$ or any other velocity. The superluminal quantum’s helical movement is what creates the
longitudinal speed $c$ of this right circularly polarized photon in a particular reference frame. The helical trajectory is not a physical object but the path of the superluminal quantum in this reference frame. In another reference frame, the helical path in that reference frame would also be stationary, and the superluminal quantum would move in the $z$ direction also with speed $c$, but its angular velocity $\omega$ and wave number $k$ would have changed by the Doppler shift that affects superluminal quanta, by their postulated characteristics, in the same way as photons.

For a real photon that is many wavelengths long, for example 100 or 1000 wavelengths long, the superluminal quantum model would show the superluminal quantum moving along a helical trajectory that is relatively stationary in the observer’s reference frame, but the front and rear end of the photon would move forward with speed $c$. The photon’s superluminal quantum would travel along its helical trajectory somewhere between the front end and the rear end of the photon, keeping its average position relative to the photon as a whole.

How does the slope of the $x$ and $y$ components of the superluminal quantum’s helical trajectory change with position along the $z$-axis? Taking the first derivatives of $x$ and $y$ with respect to $z$ in equation (11), we get

$$
\frac{dx(z)}{dz} = -\sin\left(\frac{2\pi}{\lambda} z\right) \tag{14}
$$
$$
\frac{dy(z)}{dz} = \cos\left(\frac{2\pi}{\lambda} z\right)
$$

Note that the slopes of both the $x$ and $y$ helical projections varies from $-1$ to $+1$ over one wavelength, but the $x$ and $y$ slopes are $90^\circ$ out of phase. This slope variation over a wavelength of the photon superluminal quantum’s helical trajectory is independent of the photon’s wavelength.

When we take the second derivative of $x$ and $y$ with respect to $z$, we get

$$
\frac{d^2 x(z)}{dz^2} = -\frac{2\pi}{\lambda} \cos\left(\frac{2\pi}{\lambda} z\right) \tag{15}
$$
$$
\frac{d^2 y(z)}{dz^2} = -\frac{2\pi}{\lambda} \sin\left(\frac{2\pi}{\lambda} z\right)
$$

Comparing (11) and (15) and combining them, we get
\[ \frac{d^2 x(z)}{dz^2} = -\left(\frac{2\pi}{\lambda}\right)^2 x(z) = -k^2 x(z) \] (16)
\[ \frac{d^2 y(z)}{dz^2} = -\left(\frac{2\pi}{\lambda}\right)^2 y(z) = -k^2 y(z) \]

since \( k = \frac{2\pi}{\lambda} \). For a superluminal quantum, \( p = \hbar k \) so equation (16) becomes

\[ \frac{d^2 x(z)}{dz^2} = -\frac{p^2}{\hbar^2} x(z) \]
\[ \frac{d^2 y(z)}{dz^2} = -\frac{p^2}{\hbar^2} y(z) \] (17)

where \( p \) is the longitudinal component of the superluminal quantum’s momentum, i.e. \( p \) is the photon’s longitudinal momentum. Equation (17) relates the dynamical measure \( p^2 \) to the second derivative of the helical structure of the photon. It is also a differential equation for \( x(z) \) and \( y(z) \), whose solutions, given that \( p = \hbar k \), are equation (11) for the helical structure of the photon.

Now let us consider equation (13) for the position of the superluminal quantum on its helical trajectory over time. Taking the time derivatives of \( x(t) \), \( y(t) \) and \( z(t) \) we get

\[ \frac{dx(t)}{dt} = -\frac{\lambda \omega}{2\pi} \sin(\omega t) = -c \sin(\omega t) \]
\[ \frac{dy(t)}{dt} = \frac{\lambda \omega}{2\pi} \cos(\omega t) = c \cos(\omega t) \]
\[ \frac{dz(t)}{dt} = c \] (18)

since \( \omega/2\pi = f \) and \( \lambda f = c \). Equation (18) becomes

\[ V_x(t) = \frac{dx(t)}{dt} = -c \sin(\omega t) \]
\[ V_y(t) = \frac{dy(t)}{dt} = c \cos(\omega t) \]
\[ V_z(t) = \frac{dz(t)}{dt} = c \] (19)

The total speed of the superluminal quantum along its helical path for the photon model is therefore
This gives the superluminal speed of the superluminal quantum that we found earlier in this article for movement of a superluminal quantum on a straight-axis helical path. Here the result was straightforward, but if the helix is closed as in the case of the electron so the geometry is not as simple, this way of calculating the total velocity of the superluminal quantum will prove useful.

Differentiating equation (18) once more with respect time gives

$$\begin{align*}
\frac{d^2 x(t)}{dt^2} &= -c \omega \cos(\omega t) = -\omega^2 x(t) \\
\frac{d^2 y(t)}{dt^2} &= -c \omega \sin(\omega t) = -\omega^2 y(t) \\
\frac{d^2 z(t)}{dt^2} &= 0
\end{align*}$$

(21)

The third equation above indicates that the projected speed of the superluminal quantum onto the axis of the helical trajectory has no acceleration. It remains $c$ while the superluminal quantum circulates along the helical trajectory at the speed $1.414..c$.

If we insert the photon superluminal quantum’s energy relation $E = \hbar \omega$ and the expressions for $x(t)$ and $y(t)$ from equation (13) into equation (21) we get

$$\begin{align*}
\frac{d^2 x(t)}{dt^2} &= -\frac{cE}{\hbar} \cos\left(\frac{E}{\hbar} t\right) = -\left(\frac{E}{\hbar}\right)^2 x(t) \\
\frac{d^2 y(t)}{dt^2} &= -\frac{cE}{\hbar} \sin\left(\frac{E}{\hbar} t\right) = -\left(\frac{E}{\hbar}\right)^2 y(t) \\
\frac{d^2 z(t)}{dt^2} &= 0
\end{align*}$$

(22)

This indicates that the maximum acceleration of the superluminal quantum in the transverse direction to the direction of the photon’s propagation is proportional to the
square of the photon’s energy, and that this acceleration of the superluminal quantum is always directed towards the helical axis.

Equations (17) and (22) can be summarized by the second-order differential equations

\[
\frac{d^2 x(z(t))}{dz^2} = -k^2 x(z(t)) \tag{23}
\]

and

\[
\frac{d^2 y(z(t))}{dz^2} = -k^2 y(z(t))
\]

and

\[
\frac{d^2 x(z(t))}{dt^2} = -\omega^2 x(z(t)) \tag{24}
\]

\[
\frac{d^2 y(z(t))}{dt^2} = -\omega^2 y(z(t))
\]

which in the complex notation form (10) becomes

\[
\frac{d^2 R(z(t))}{dz^2} = -k^2 R(z(t)) \tag{25}
\]

and

\[
\frac{d^2 R(z(t))}{dt^2} = -\omega^2 R(z(t)) \tag{26}
\]

If we solve equation (26) for \( R(z(t)) \) and substitute this into equation (25) we get

\[
\frac{d^2 R(z(t))}{dz^2} = \frac{k^2}{\omega^2} \frac{d^2 R(z(t))}{dt^2} \tag{27}
\]

\[
= \frac{1}{c^2} \frac{d^2 R(z(t))}{dt^2}
\]

This differential equation is the wave equation. Its solution is waves traveling at the speed \( c \), one of whose solutions is the trajectory of the photon’s superluminal quantum in equation (10)

\[
R(z(t)) = \frac{\lambda}{2\pi} e^{ikz(t)} \tag{28}
\]

\[
z(t) = ct
\]

Note that the photon superluminal quantum’s trajectories that are solutions to the wave equation have amplitudes whose dimensions are length. In electromagnetic theory,
the amplitudes of solutions of the wave equation are electric and magnetic field
amplitudes. The real function $R^* R$ is given by

$$R^* R = \left( \frac{\lambda}{2\pi} \right)^2$$

is the square of the amplitude of the complex function $R$. In quantum mechanics this
amplitude, when normalized over a particular region of $R$, represents the probability of
finding a photon within a particular region of the function $R$.

If certain boundary conditions were set on the solutions in equation (27), such as
the photon is reflected back and forth between two parallel mirrors so that only photons
that are in phase with itself after reflecting off of both mirrors are able to remain between
the mirrors, there would be only certain discrete superluminal quantum trajectory
solutions to the above second-order differential equations. So the wave equation for
photon superluminal quanta will then have discrete eigenfunction solutions, with discrete
sets of eigenvalue wave numbers $k_n$ corresponding to these eigenfunctions. Each
eigenfunction would correspond to a particular helical trajectory of a superluminal
quantum, which would have the corresponding eigenvalue wave number $k$.

The superluminal quantum model of the electron

The superluminal quantum model of the electron will be described below. The
superluminal quantum model is found to be consistent with the small and rapid
oscillatory motion “Zitterbewegung” of the electron that is predicted by the Dirac
equation and comes into the theory of quantum electrodynamics. The model is also
consistent with the electron’s spin and the value of its magnetic moment that come out of
solutions to the Dirac equation.

“Zitterbewegung”(ZBG) means “trembling motion”. The term was first used
by Schrödinger (1930) to describes the rapid, speed-of-light oscillatory motion of the
electron in its rest frame. No internal spatial structure of the electron has been observed
experimentally. High energy electron scattering experiments have put an upper value on
the electron’s size at about $10^{-18}$ meters. Yet Schrödinger's ZBG results from the Dirac
equation suggest that the electron’s rapid oscillatory motion has a radius of
$R_{\text{pit}} = \frac{\sqrt{2}}{2} \frac{\hbar}{m c}$ or $1.9 \times 10^{-13}$ meters. Furthermore, the electron’s instantaneous speed is $c$, and it has an associated frequency of $\omega_{\text{pit}} = 2mc^2/\hbar$, twice the angular frequency of a photon whose energy is that contained within the rest mass of an electron. The apparent contradiction between these experimental and theoretical results has been problematical for creating an acceptable spatial model for the electron.

**Equations of motion for the electron’s superluminal quantum**

The superluminal quantum model of the electron describes the electron as a superluminal quantum moving along a closed, double-looped helical trajectory in the electron model’s rest frame, that is, the frame where the superluminal quantum’s trajectory closes on itself. (If the electron is moving, the superluminal quantum’s double-looped helical trajectory will not close on itself.) The superluminal quantum’s trajectory’s closed helical axis’ radius is $R_{0} = \frac{1}{2} \frac{\hbar}{m c}$ and the helical radius is $R_{\text{helix}} = \sqrt{2}R_{0}$. In comparison with the superluminal quantum photon model, the superluminal quantum electron model structurally resembles a superluminal quantum photon model of angular frequency $\omega_{0} = mc^2/\hbar$, wavelength $\lambda_{c} = h/mc$ (the Compton wavelength) and wave number $k_{0} = 2\pi / \lambda_{c}$ that makes a double-looped helical trajectory having a circular axis of circumference $\lambda_{c}/2$. After following its helical trajectory around this circle once, the superluminal quantum’s trajectory is $180^\circ$ out of phase with itself and doesn’t close on itself, but after the superluminal quantum follows its helical trajectory around the circle a second time, the trajectory is back in phase with itself and closes upon itself. The total distance along its circular axis that the circulating electron superluminal quantum has traveled before the superluminal quantum trajectory closes is $\lambda_{c}$. Like a photon’s superluminal quantum whose wavelength is $\lambda_{c}$, the electron’s superluminal quantum carries longitudinal momentum $p = h/\lambda_{c}$ directed along its trajectory, as well as transverse components of momentum, and energy $E = \hbar \omega_{0}$. However, unlike the photon’s superluminal quantum which is uncharged, the electron’s superluminal quantum carries the electron’s negative charge $-e$.

So the spatial trajectory for the superluminal quantum in the electron model is
\[ x(d) = R_0(1 + \sqrt{2} \cos(2\pi d / \lambda_c)) \cos(4\pi d / \lambda_c) \]
\[ y(d) = R_0(1 + \sqrt{2} \cos(2\pi d / \lambda_c)) \sin(4\pi d / \lambda_c) \]  
\[ z(d) = R_0\sqrt{2} \sin(2\pi d / \lambda_c) \]  

(30)

where \( R_0 = \frac{1}{2} \hbar / mc \) is the radius of the circle which is the axis of the double-looped helix, and \( d \) is the distance forward along the circular axis that the superluminal quantum has moved while following its helical trajectory. This forward speed is postulated to be \( c \), so for the traveling superluminal quantum, \( d = ct \).

Figure 2. Modeling an electron. Two views of the superluminal quantum's closed double-looped helical trajectory. The circle in the x-y plane of radius \( R_0 = \frac{1}{2} \hbar / mc = 1.9 \times 10^{-13} \) meters, is the axis of the closed helix. The maximum speed of the superluminal quantum in the electron’s rest frame is 2.797..\( c \).

Note that when \( d = \frac{1}{2} \lambda_c \), the term \( 2\pi d / \lambda_c \) above has value \( \pi \) and so \( \cos(2\pi d / \lambda_c) \) and \( \sin(2\pi d / \lambda_c) \) are only 180° into their cycle, which only reaches \( 2\pi \) when \( d = \lambda_c \) (at two rotations around the loop). The second term \( 4\pi d / \lambda_c \) is in phase at both \( d = \frac{1}{2} \lambda_c \) which gives the phase \( 2\pi \) (in phase), and \( d = \lambda_c \) which gives the phase \( 4\pi \) (in phase), but all of the sine and cosine terms in the equations above have to be in phase for the trajectory to close on itself.

Since \( d = ct \), the term \( \cos(2\pi d / \lambda_c) \) from (30) becomes
\[ \cos(2\pi d / \lambda_c) = \cos(2\pi ct / \lambda_c) = \cos(2\pi f_d t) = \cos(\omega_d t) \]  
(31)

and similarly for the other terms in (30). So the position with time of the superluminal quantum in the electron model is
\[ x(t) = R_0(1 + \sqrt{2} \cos(\omega_0 t)) \cos(2\omega_0 t) \]
\[ y(t) = R_0(1 + \sqrt{2} \cos(\omega_0 t)) \sin(2\omega_0 t) \]
\[ z(t) = R_0\sqrt{2} \sin(\omega_0 t) \]

(32)

In the above equation, at \( t = 0 \), the superluminal quantum is at \( \left(x = R_0(1 + \sqrt{2}), y = 0, z = 0\right) \). According to the double looping photon comparison, the above equations correspond to a left-circularly polarized photon of wavelength \( \lambda_c \), traveling counterclockwise in a closed double loop (as seen from the +z direction in the x-y plane).

Properties the superluminal quantum model of the electron shares with the Dirac equation’s electron with Zitterbewegung

1. The calculated electron spin \( s = \frac{1}{2} \hbar \).
2. The calculated electron magnetic moment \( \mu = e \hbar / 2mc \) the Bohr magneton (\( g = 2 \)).
3. The Zitterbewegung light velocity \( c \) of the electron (the electron model resembles a circling photon model with speed \( c \)).
4. The Zitterbewegung frequency \( 2\omega_0 \), where \( \omega_0 = mc^2 / \hbar \).
5. The Zitterbewegung radius \( R_0 = \frac{\hbar}{2mc} \).
6. The Zitterbewegung distinction between the coordinate of the electron’s charge and the position of the electron as a whole.
7. The prediction of an electron and a positron (the two possible helicities do this in the superluminal quantum model).
8. The non-conservation of momentum of the Zitterbewegung motion.
9. The electron’s total motion is the sum of its linear motion and its circulatory motion.

Explaining the electron’s Zitterbewegung

Let us see how the superluminal quantum’s motion described in \((32)\) compares with the Zitterbewegung motion of the electron described by Schrödinger:
1) **Zitterbewegung** has an internal frequency of $\omega_{\text{zitt}} = 2mc^2 / h = 2\omega_0$.

The superluminal quantum’s trajectory above is defined by both the frequencies $\omega_0$ and $2\omega_0$, where $\omega_0 = mc^2$. $\omega_0$ is the angular frequency of a photon whose energy is $mc^2$.

2) **Zitterbewegung** has an internal radius $R_{\text{zitt}} = \frac{1}{2} h / mc$.

The superluminal quantum has $R_0 = \frac{1}{2} h / mc = R_{\text{zitt}}$ as the radius of the axis its closed, double-looped helical trajectory.

3) **Zitterbewegung**: The speed of light is associated with the electron’s motion since the Dirac equation has eigenvalue solutions of $\pm c$ for the electron (and the positron). The speed of forward movement of the superluminal quantum along its helical was postulated to be $c$ for both the electron model and the photon model. This is incorporated into the superluminal quantum trajectory equations for the electron model as $d = ct$, where $d$ is the forward distance along the helical axis traveled by the superluminal quantum as it follows its helical trajectory. The speed of the superluminal quantum itself along its trajectory is however superluminal as it also is in the photon model, in order for the superluminal quantum to keep up with the speed-of-light forward movement along its trajectory’s circular axis. As can be calculated from the superluminal quantum photon and electron models, the maximum speed of the superluminal quantum in the photon model is $1.414..c$ as previously noted, while in the electron model, the superluminal quantum’s maximum speed in the superluminal quantum electron’s rest frame is $2.797..c$.

Other properties of the superluminal quantum model of the electron:

1) **Prediction of the electron’s antiparticle.** The two possible helicities of the superluminal quantum’s closed helical path correspond to an electron and a positron. (The positron was first predicted from the Dirac equation’s two speed-of-light eigenvalue solutions). By reversing the helicity for the superluminal quantum described by the above closed double-looped helix, whose left circulating superluminal quantum corresponds to a left-circularly-polarized photon, the corresponding anti-particle’s superluminal quantum
would be formed, which would correspond to a circulating right circularly polarized photon. A positive charge of +e would have be supplied to the positron’s superluminal quantum for symmetry. The superluminal quantum electron model does not however predict whether the circulating left-turning superluminal quantum trajectory described by equation (32) is actually associated with an electron or a positron. It should be possible to determine this by an experiment.

The relationship between charge and helicity in the superluminal quantum’s electron and photon models is suggestive of a deeper relationship between helicity, spin and charge. The superluminal quantum photon model has an open helix trajectory and spin $\hbar$ and no charge. The superluminal quantum electron model has a closed double-looped helical trajectory, spin $\frac{1}{2}\hbar$ and a negative charge (positive for a positron). Are there any elementary particles with rest mass that could be described by a closed single-looped helix whose circumference is 1 wavelength. Its spin would be $\hbar$, but what would be its charge?

2) The spin of the electron. The value of $R_0$, the superluminal quantum electron model’s trajectory in equation (30) is chosen to obtain the electron’s know value of spin $\hbar/2$, the value found from the Dirac equation.

The formula for angular momentum of a spinning or circulating objects with momentum $\vec{P}$ at a distance $\vec{R}$ from the rotational axis is $\vec{S} = \vec{R} \times \vec{P}$. In the superluminal quantum model for the electron, the superluminal quantum’s instantaneous position and momentum can be described by a radial vector $\vec{R}$ and a momentum vector $\vec{P}$ whose magnitudes are related to its position along its trajectory and to the curvature of its trajectory (which corresponds to a wavelength and an associated momentum $p = h/\lambda$).

$\vec{R}$ and $\vec{P}$ are constantly changing as the superluminal quantum moves along its closed double-looped helical trajectory. The magnitude of $\vec{P}$ is also not constant in the electron model, because the instantaneous magnitude of $\vec{P}$ depends on the immediate curvature of the trajectory where the superluminal quantum is located, and that curvature is not constant for a closed helical trajectory. As the magnitude of the curvature decreases, giving a lower magnitude of $\vec{P}$ as the radial component of $\vec{R}$ from the z-axis increases,
the trajectory curvature decreases, giving a lower value of the magnitude of $\vec{P}$. A geometrical analysis of the model finds that the magnitude of the superluminal quantum’s transverse component of $\vec{P}$ around the z-axis varies inversely with the magnitude of the superluminal quantum’s radial distance from the z-axis. The result is that the spin product $RP$, where $R$ is the radial distance from the z-axis to the superluminal quantum, and $P$ is the component of $\vec{P}$ that is in the x-y plane and perpendicular $R$, is independent of the position of the superluminal quantum along its trajectory. So the value of the z component of $\vec{S}$, that is, $S_z = RP$ can be calculated using

$$R = R_0 = \frac{1}{2} \hbar / mc$$

and

$$P = 2\pi \hbar / \lambda = 2\pi \hbar / \lambda_c = 2\pi \hbar / (2\pi \hbar / mc) = mc$$

Therefore the average value of spin for the superluminal quantum model of the electron is

$$S_z = RP$$

$$= \left(\frac{1}{2} \hbar / mc\right)(mc)$$

$$= \frac{1}{2} \hbar$$

which is the value of spin of the electron found from the Dirac equation, and which is experimentally accurate also.

3) The magnetic moment of the electron. The superluminal quantum electron’s magnetic moment’s value is set to be the Dirac value of $M_z = -e\hbar / 2m$ by selecting the value of the radius of the superluminal quantum’s closed helical trajectory to be $\sqrt{2}R_0$ where $R_0 = \frac{1}{2} \hbar / mc$.

The magnetic moment component $M_z$ in the superluminal quantum model of the electron is calculated below. The $M_x$ and $M_y$ components vanish by symmetry, when averaged over the different possible orientations of the closed double looped helical trajectory around the z-axis.

The magnetic moment component $M_z$ of a charge $q$ moving with a constant speed $V$ in a circle in the x-y plane of radius $R$ is $M_z = \frac{1}{2} qVR$. When we are given the position $R$
and velocity \( V \) of a charged particle moving along a particular closed path, with \( x \) and \( y \) position and velocity coordinates given by \( R_x(\theta), V_x(\theta), R_y(\theta) \) and \( V_y(\theta) \) with \( \theta \) going from 0 to \( 2\pi \) around the closed path, \( M_z \) is found from

\[
M_z = \frac{1}{4\pi} \int_{0}^{2\pi} q \left( R_x(\theta)V_y(\theta) - R_y(\theta)V_x(\theta) \right) d\theta
\]

The value \( \sqrt{2} \) was selected as a measure of the radius \( R_{\text{helix}} = \sqrt{2}R_0 \) of the double-looped helix of the electron model, because when this value is inserted into equation (36) using the 3 spatial component equations for the electron model, the result is that for the superluminal quantum electron model

\[
M_z = -\frac{eh}{2m}
\]

This is the \( g = 2 \) value of the magnetic moment of the electron found from the Dirac equation.

If in equation (36) for \( M_z \), 0 is used as a measure of the helix radius instead of \( \sqrt{2} \) (implying that the charge \(-e\) is always circulating at the same radius, then it is found that \( M_z = -\frac{eh}{4m} \) which is the classical value of \( M_z \) for a charge of \(-e\) traveling in a circle at the speed of light at a radius \( R = R_0 = \frac{1}{2}h/mc \), i.e. this is the classical \( g = 1 \) value of the magnetic moment to spin relationship. So the presence of the double-looped helical trajectory with helical radius \( R_{\text{helix}} = \sqrt{2}R_0 \) has the effect of doubling the calculated magnetic moment of the superluminal quantum, compared with the classical \( g = 1 \) electron model, without changing the superluminal quantum electron model’s calculated spin.

The Schrödinger equation and the superluminal quantum electron model

We found that quantitative consideration of the trajectory of the superluminal quantum photon model led to equations resembling the Schrödinger equation. If the superluminal quantum model of the electron is basically a correct representation of the electron, then the model should lead to the Schrödinger equation for the electron, which for one special dimension in the \( z \) direction is
\[ -\hbar^2 \frac{d^2 \Psi(z,t)}{dz^2} + V(z) \Psi(z,t) = i\hbar \frac{d\Psi(z,t)}{dt} \]  \hspace{1cm} (37)

Can we obtain the Schrödinger equation with the superluminal quantum model, starting out as Schrödinger did with kinetic energy plus potential energy equals total energy:

\[ \frac{p^2}{2m} + V(z) = E \]  \hspace{1cm} (38)

For a free particle where \( V(z) = 0 \), are generally proposed in the form of a plane wave

\[ \Psi(z,t) = Ae^{ikz+ikt} \]  \hspace{1cm} (39)

But for a superluminal quantum following a helical trajectory of radius \( R_0 \), the superluminal quantum’s position coordinates as a function of \( z \) and \( t \) are given in complex exponential form as

\[ R(z(t)) = R_0 e^{ikz(t)} \]
\[ z(t) = Vt \]  \hspace{1cm} (40)

Compare this equation for an electron with equation (10) for a photon. Equation (40) corresponds to a superluminal quantum traveling along an open helical trajectory of radius \( R_0 \) with a longitudinal velocity \( V \).

\[ x(z(t)) = R_0 \cos(kz(t)) \]
\[ y(z(t)) = R_0 \sin(kz(t)) \]
\[ z(t) = Vt \]  \hspace{1cm} (41)

where \( e^{ikz(t)} = \cos(kz(t)) + i\sin(kz(t)) \) in the conventional sense. For an electron, the relations among \( p \), \( E \), \( k \), \( \omega \) and \( V \) are given by

\[ p = \hbar k \]
\[ E = \hbar \omega \]
\[ k = \frac{2\pi}{\lambda} \]
\[ V = \frac{\omega}{k} \]  \hspace{1cm} (42)

where \( V \) is the phase velocity of the helical wave movement.

Can \( R(z,t) \), the position of the helically moving electron superluminal quantum, lead to a Schrödinger-like equation for the electron? Multiplying all terms in equation (38) by \( R(z(t)) \) gives
\[
\frac{p^2}{2m} R(z(t)) + V(z)R(z(t)) = ER(z(t))
\] (43)

Taking the first and second derivatives of \( R(z(t)) \) gives

\[
\frac{dR(z(t))}{dz} = ikr_0 e^{ikz(t)}
\]

(44)

and

\[
\frac{d^2R(z(t))}{dz^2} = -k^2 r_0 e^{ikz(t)}
\]

(45)

Substituting \( k = \frac{p}{\hbar} \) into equation (45), rearranging terms, and dividing both sides by \( 2m \) gives

\[
\frac{d^2R(z(t))}{dz^2} = -\frac{p^2}{\hbar^2} R(z(t))
\]

(46)

Taking the first derivative of \( R(z(t)) \) in equation (40) with respect to time using the chain rule gives

\[
\frac{dR(z(t))}{dt} = \frac{dR(z(t))}{dz} \times \frac{dz(t)}{dt} = ikr_0 e^{ikz(t)} \times V
\]

(47)

Substituting \( \omega = \frac{E}{\hbar} \) into equation (47) and rearranging gives
\[
\frac{dR(z,t)}{dt} = \frac{iE}{\hbar} R(z,t)
\]

\[
\frac{\hbar dR(z,t)}{dt} = ER(z,t)
\]

(48)

\[-i\hbar \frac{dR(z,t)}{dt} = ER(z,t)\]

Comparing equations (46) and (48) with equation (43), we see that for a free particle, where \( V(z) = 0 \), we get

\[
\frac{p^2}{2m} R(z,t) + V(z) R(z,t) = ER(z,t)
\]

\[
\frac{p^2}{2m} R(z,t) = ER(z,t)
\]

(49)

\[-\frac{\hbar^2}{2m} \frac{d^2R(z,t)}{dz^2} = -i\hbar \frac{dR(z,t)}{dt}\]

Compare this result with the Schrödinger equation (37) for a free electron

\[
\frac{-\hbar^2}{2m} \frac{d^2\Psi(z,t)}{dz^2} = i\hbar \frac{d\Psi(z,t)}{dt}
\]

(50)

The difference in the sign of the right hand sides of equations (49) and (50) is due to the plane wave expression for \( \Psi \) in equation (39)

\[\Psi(z,t) = Ae^{ikz - i\omega t}\]

(51)

as compared with the coordinates for the position of the superluminal quantum along its helical trajectory given by equation (40)

\[R(z(t)) = R_0 e^{iz(t)}\]

\[z(t) = Vt = \frac{\omega}{k} t\]

(52)

In the one-dimensional case, where there is a potential \( V(z) \) in which the electron can move, the Schrödinger-like trajectory equation for the superluminal quantum becomes

\[-\frac{\hbar^2}{2m} \frac{d^2R(z(t))}{dz^2} + V(z) R(z(t)) = -i\hbar \frac{dR(z(t))}{dt}\]

(53)

which corresponds to the 1-dimensional Schrödinger equation (37)
If equation (51) for a plane wave moving in the +z direction had been written
\[ \Psi(z,t) = Ae^{i\omega t - ikz} \] instead of \[ \Psi(z,t) = Ae^{-ikz - i\omega t} \], then the Schrödinger equation in (54)
would instead be written with \(-i\) instead of \(i\) in the right hand term as
\[ \frac{-\hbar^2}{2m} \frac{d^2\Psi(z,t)}{dz^2} + V(z)\Psi(z,t) = -i\hbar \frac{d\Psi(z,t)}{dt} \] (55)
which more precisely matches the Schrödinger-like trajectory equation (53) for the superluminal quantum.

How does \( R_0 \), the radius of the superluminal quantum’s helical trajectory used in obtaining the superluminal quantum’s Schrödinger-like equation for the electron, relate to the wavelength \( \lambda \) of the electron? In the superluminal quantum model of the photon, the relation of the photon’s radius to its wavelength was found to be \( R = \frac{\lambda}{2\pi} \) by combining the equations for the photon’s momentum and spin with the proposed flow of the photon’s momentum along a helical trajectory. A similar calculation can be done for the electron, where the only difference is the electron’s spin is \( \frac{1}{2} \hbar \) as compared to the photon’s spin \( \hbar \). The result of the calculation for the superluminal quantum’s radius for the moving electron with wavelength \( \lambda \) is
\[ R_0 = \frac{\sqrt{2}}{2\pi} \lambda \geq \frac{.707}{2\pi} \lambda \] (56)
So for the superluminal quantum electron’s Schrödinger-like equation, the radius of the superluminal quantum’s helix for a moving electron is .707 times that of an electron having the same wavelength, and this is because the electron has half the spin of the photon. Using this result, equation for the superluminal quantum’s trajectory for a free electron (40) becomes
\[ R(z,t) = \frac{\sqrt{2}\lambda}{2\pi} e^{i\lambda t} \] (57)
that is,
\[
\begin{align*}
x(z,t) &= \frac{\sqrt{2}\lambda}{2\pi} \cos(kz(t)) \\
y(z,t) &= \frac{\sqrt{2}\lambda}{2\pi} \sin(kz(t)) \\
z(t) &= Vt
\end{align*}
\] (58)

The superluminal quantum trajectory equation (57) can be understood in completely real terms as equation (58). For example, at \( t = 0 \) and \( z = 0 \) in the superluminal quantum trajectory defined by equation (41), the first term of the Schrödinger-like trajectory equation (53) indicates that when the magnitude of the rate of change with \( z \) of the slope of the \( x \) component of the superluminal quantum’s trajectory is maximum, i.e. when \( \cos(kz) = 1 \), the first derivative term on the right indicates that the magnitude of the rate of change of the \( x \) component of the position of the superluminal quantum, i.e. the magnitude of its component of speed in the \( x \) direction, is minimum since \( \sin(\omega t) = 0 \).

Whereas a solution \( \Psi(z,t) = A e^{ikz-i\omega t} \) to the Schrödinger equation for a free electron in this example gives \( \Psi^\ast \Psi = A^2 \) as the constant probability density (when \( \Psi^\ast \Psi \) is normalized over a particular region) of finding an electron between two unit \( z \) intervals along the \( z \)-axis, the value \( R(z,t) = \frac{\sqrt{2}\lambda}{2\pi} e^{ikz(t)} \) gives \( \Psi^\ast \Psi = \frac{\lambda^2}{2\pi} \) for square of the magnitude of the constantly rotating radius vector to the position of the electron’s superluminal quantum along its trajectory.

If a potential \( V(x) \) causes the electron’s superluminal quantum to gain or lose momentum, the magnitude of the radius of its helical trajectory will decrease or increase accordingly. A larger trajectory radius indicates a slower, lower momentum superluminal quantum and a greater likelihood of the superluminal quantum being found where its speed is slower, as compared to where its helical radius is smaller and its speed is faster.

The relativistic superluminal quantum electron picture is more complex than described above. In that case, the superluminal quantum is travelling superluminally at the Zitterbewegung frequency, and generating the de Broglie wavelength through self-interference of its possible trajectory wave patterns as the electron as a whole moves with momentum \( p \) in the \( z \) direction. The Schrödinger-like trajectory equation is non-
relativistic and doesn’t take the superluminal quantum electron’s internal energy, frequency and closed double-helical trajectory into consideration. In the relativistic case, the Dirac equation provides possible superluminal quantum trajectories that include the electron’s *Zitterbewegung* and its spin and magnetic moment which are not included in the non-relativistic Schrödinger-like trajectory equation analysis.

**Other aspects of the superluminal quantum models**

**Binding force on superluminal quantum.** A centrally directed force would be necessary to hold the superluminal quantum in its helical orbit. That force cannot come from other superluminal quanta since there is only one superluminal quantum per electron or photon. An initial calculation of this binding force on the superluminal quantum in a photon shows that the size of this binding force for a photon with the Compton wavelength $\lambda_c$ is $F = \frac{mc^2}{\lambda_c} = .216$ newtons, quite a large force within a photon.

The force holding the electron model together is twice that due to the double looped closed helix which reduces the electron’s size. And the binding force on a proton’s superluminal quantum (if it had the same superluminal quantum structure as an electron—which it doesn’t due to it quark structure) would be $(1833)^2$ or about 3.3 million times as large as the binding force on an electron’s superluminal quantum.

**Interactions between superluminal quanta**

According the superluminal quantum model, photons and electrons are formed by circulating superluminal superluminal quanta, and the elementary particle is a kind of time and spatial average of the movement of the superluminal quantum. However, in electron scattering experiments, it is the superluminal quantum from one electron which scatters the superluminal quantum of a high energy superluminal quantum, so that the size measures found for the electron seem to be point-like (maximum size less than $10^{-18} m$ so far). According to the superluminal quantum model, the superluminal quantum has no extension in space at all. Its movement along different helical trajectories with a charge of $-e$ or $+e$ for the closed double looped trajectory of an electron or positron, and
with no charge for the open helical trajectory of a photon. Whenever an elementary particle transfers its energy and/or momentum to another particle, it is the point-like superluminal quantum, the carrier of the energy, momentum and charge in an elementary particle, which transfers energy and momentum to the superluminal quantum of the other particle in a highly localized region.

**Circulating momentum and the electron’s mass**

The above description of the electron gives the momentum \( p_o = mc \) a role in the structure of an electron than has been previously overlooked. Consider the Einstein-de Broglie relation applied to an electron,

\[
E^2 = p^2 c^2 + m^2 c^4
\]  

(59)

where \( p \) is the magnitude of the electron’s linear momentum and \( E \) is the electron’s total energy. Replacing \( m \) by \( p_o / c \) in the equation gives

\[
E^2 = p^2 c^2 + p_o^2 c^2
\]  

(60)

This equation is more revealing that the previous one because it indicates the deeper nature of the electron as a packet of circulating momentum \( p_o = mc \). Furthermore, if we insert \( p^2 = p_x^2 + p_y^2 + p_z^2 \) into the above equation we get

\[
E^2 = (p_x^2 + p_y^2 + p_z^2)c^2 + p_o^2 c^2
\]  

\[= (p_x^2 + p_y^2 + p_z^2 + p_o^2)c^2\]  

(61)

indicating that \( p_o \) can be considered to be orthogonal in a mathematical sense with \( p_x \), \( p_y \) and \( p_z \).

Both spin and charge are related to helical trajectories in the superluminal quantum model and are fundamental common features of the elementary particles. The double-looped helical structure of the electron superluminal quantum’s trajectory apparently gives rise to the electron’s superluminal quantum’s charge and therefore the electron’s charge. The superluminal quantum concept does not have to be limited to modeling photons and electrons. I propose that all elementary particles, including quarks, gluons and neutrinos, may be modeled as individual pointlike superluminal quanta (with one superluminal quantum per elementary particle except for composite particles like
protons and pions) which transport momentum and energy superluminally in closed or open helical trajectories. Elementary particles travel subluminally when the helices are closed (or nearly closed for moving particles), and at the speed of light when the helices are linear and open.

Furthermore, in the superluminal quantum model of elementary particles no superluminal quantum is ever at rest since superluminal quanta always move superluminally, yet some elementary particles are associated with a rest mass. I propose that superluminal quanta with superluminal closed and open helical trajectories compose all elementary particles. An elementary particle with rest mass can be said to be in its rest frame when its superluminal quantum’s double-looped helical path exactly closes upon itself. This can happen with particles with rest mass but not with photons or gluons which have zero rest mass.

**Testing the superluminal quantum hypothesis**

Testing for the existence of superluminal particles that circulate approximately $2 \times 10^{20}$ times per second to form electrons will require great ingenuity. Yet this is the electron’s *Zitterbewegung* frequency that comes directly from solutions to the relativistic Dirac equation. Dirac conceived of the electron as a charged point-like particle whose movement forms the electron’s physical characteristics such as its spin and magnetic moment. I have called this point particle a superluminal quantum because similar point particles with different trajectories and charge may also compose the photon and all the other elementary particles. It is a superluminal quantum’s helical trajectory that determines what kind of elementary particle the superluminal quantum forms—different trajectories correspond to different particles and several superluminal quanta with their trajectories would form compound particles like protons.

It might be objected that since superluminal quanta always travel superluminally, they violate the relativistic upper limit of $c$ on the transport of information. But the elementary particles that superluminal quanta form never travel faster than $c$, so this is not a valid objection to the possible existence of superluminal quanta.

Since superluminal quanta also form photons which can have frequencies much lower than $2 \times 10^{20}$ hertz, one possible approach to testing the the superluminal quantum
hypothesis is to detect them in low frequency photons, possibly in the frequency range of visible light or microwave radiation. Laser or maser radiation is highly coherent and this could facilitate the localization of helically moving superluminal quanta in individual photons that are all moving in phase.

Here is another possible test. An electron and a positron differ in the direction of their helicity and resemble a circulating left or right circularly polarized photon, according to the superluminal quantum hypothesis. I predict that electrons would differentially absorb or scatter left and right circularly polarized photons having energies corresponding to the rest energy of electrons, because electrons would have internal superluminal quantum frequencies corresponding to the frequencies of the incoming circularly polarized photons.

**Conclusion: the emerging superluminal quantum picture of elementary particles**

Clear and testable superluminal quantum models of the photon and the electron are emerging. Both the photon and the electron are composed of a helically circulating superluminal superluminal quantum having both particle-like and wave-like characteristics. Superluminal quanta are proposed to be associated with the wave functions that characterize photons and electrons. That is, it is the point-like superluminal quantum whose probability of detection is described by the solutions of the equations of Maxwell, Schrödinger and Dirac.

**References**

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